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GREENBELT, MARYLAND

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ABSTRACT

The present Orbiting Astronomical Observatory (OAO-A2) is designed to point its optical axis to any location on the celestial sphere, and then maintain that pointing within one minute of arc while making astronomical observations. This is accomplished under control of six two-axis gimballed startrackers. In order to ensure this precision the startrackers must be properly aligned relative to each other and to the spacecraft structure. Since this alignment can change due to thermal and launch stresses, the alignment must be performed after the spacecraft is in orbit.

Four misalignment parameters are modeled for each startracker: the three rotational misalignments about the coordinate axes, and a null shift in the inner gimbal. Then two procedures for estimating these parameters from in-orbit telemetry are analyzed. The special procedure, which applies when the spacecraft attitude is perfectly known, involves expressing the known true direction cosines of the guide star in terms of the measured gimbal angles and the four unknown misalignment parameters. The general procedure, which applies when the spacecraft attitude is known only to be near nominal, involves considering the trackers by pairs. For each pair, the known true angle between the pair of guide stars is expressed in terms of the measured gimbal angles and the eight unknown misalignment parameters.

Some problems encountered in the use of these procedures are briefly described, and the actual misalignment parameters estimated in-orbit on the OAO-A2 are presented.

I. INTRODUCTION

The Orbiting Astronomical Observatory (OAO) is designed to maintain precise pointings in space within one minute of arc under the control of six two-axis gimballed startrackers. Such precision cannot be maintained unless the startrackers are aligned relative to one another and to the spacecraft structure to the same order of magnitude. Launch vibration and thermal stresses cause alignment deviations which must be calibrated out while the spacecraft is in orbit.

In this paper, the mathematical analysis of techniques for performing this calibration on the OAO is developed. Then some problems peculiar to the OAO are discussed briefly, and actual misalignment parameters estimated in-orbit on the OAO-A2 are presented.

II. GENERAL DISCUSSION

The Orbiting Astronomical Observatory (OAO) was launched successfully on December 7, 1968. Two days later, its six gimballed startrackers were turned on, directed through a computer-controlled star search pattern, and successfully acquired their predetermined guide stars. Thus began one of the most complex control procedures ever attempted, as the Support Computer Program System (SCPS) at Goddard Space Flight

Center took on the 24-hour-a-day task of directing and maintaining the spacecraft pointing axis to preassigned targets within one minute of arc. This is the design pointing accuracy in the coarse-pointing mode, in which control is derived from the gimballed startrackers.[1]

Any desired attitude in space can in theory be maintained simply by calculating the gimbal angles required to point two or more startrackers at predetermined noncollinear guide stars. If the trackers are then directed at these guide stars and the actual gimbal angles required to do this are observed, the error angles between the observed and computed gimbal angles can be processed to provide control signals for restoring the spacecraft to the nominal attitude.

Many practical problems arise to complicate the picture. Lack of control stability arises for some tracking configurations, especially near collinearity of two trackers. Occulting bodies are troublesome, specifically the "effective" earth, which has a diameter of 164° at the 500-mile orbital altitude of the OAO. As seen from the spacecraft, the earth revolves about the OAO once every orbital period (about 100 minutes). This makes it necessary for the SCPS to provide alternate startracker-star pairs, which are switched out of or into the control loop as various guide stars become occulted or unocculted.

The practical problem being addressed in this paper is the post-launch alignment of the gimballed startrackers. Prior to launch, the startrackers are precisely aligned, in the sense that the directions of their optical and gimbal axes are fixed in the control coordinate system. The misalignments from nominal (perfect orthogonality) are then entered into the computational model of the spacecraft in the SCPS, so that corrections to the gimbal commands can be computed which will ensure that the startrackers are accurately pointed. During launch, however, violent stresses occur which change these misalignments. On OAO-A2, these changes have been as large as five arcminutes. Even in the orbital environment, thermal stresses cause measurable changes in the misalignments. On OAO-A2, a 180° roll about the spacecraft optical axis causes changes greater than one arcminute in some misalignments. It is clear that such misalignment discrepancies are intolerable if the desired coarse pointing accuracy of one arcminute is to be maintained. Even if pointing accuracy were not a problem, pointing precision is. Data quality is greatly degraded when misaligned trackers, dropping out of or coming into the control loop as their stars become occulted or unocculted, cause large movement of the spacecraft pointing axis.

III. MATHEMATICAL MODELS

A. Notational Preliminaries

The mathematics employed in the ensuing analysis is almost exclusively the algebra of rotations, since abstractly the problem is one of locating various three-dimensional rectangular coordinate systems, with a common origin, with respect to one another. A notation of the form

$$i \xrightarrow{R} j$$

will mean that a rotation R transforms vectors from some coordinate system i to some other coordinate system j. Computationally this means that if $\vec{v}^{\,i}$ is an ordered triple representing a fixed abstract vector $\vec{v}$ in some coordinate frame i, and j is some other coordinate frame, then R is the matrix such that $\vec{v}^{\,j} = R\,\vec{v}^{\,i}$. Obviously then $\vec{v}^{\,i} = R^T\,\vec{v}^{\,j}$, where the superscript T signifies the transpose. The notation

$$i \xrightarrow[y]{\theta} j$$

will mean that the transformation from the i coordinate system to the j coordinate system is effected by rotating the i coordinate system in the positive sense (right-hand rule) about its y-axis through an angle θ, i.e.,

$$\vec{v}^{\,j} = \begin{pmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{pmatrix} \vec{v}^{\,i}$$

where $c\theta \equiv \cos\theta$, $s\theta \equiv \sin\theta$.

B. Mathematical Model of Spacecraft

The present application concerns only the relationships among directions in space; hence the OAO spacecraft will be adequately modeled by considering only its body-fixed axes, the "control" coordinate system c. This is a standard right-hand orthogonal triad of axes x_c, y_c, z_c (Fig. 1). The Wisconsin Experiment Package (WEP) experiment optics on OAO-A2 are nominally aligned to the $+x_c$-axis; the Smithsonian

[1] The follow-on spacecraft OAO-B and OAO-C will have a fine-pointing capability, in which control is derived from the experimenter's sensor. These spacecraft have design pointing accuracies in the one arc<u>second</u> range.

Astrophysical Observatory (SAO) cameras are nominally aligned to the $-x_c$-axis.

The six startrackers are oriented one at each end of the three coordinate axes, as shown in Fig. 1. Local tracker coordinate systems x_k, y_k, z_k, $k = 1, 2, \cdots, 6$,

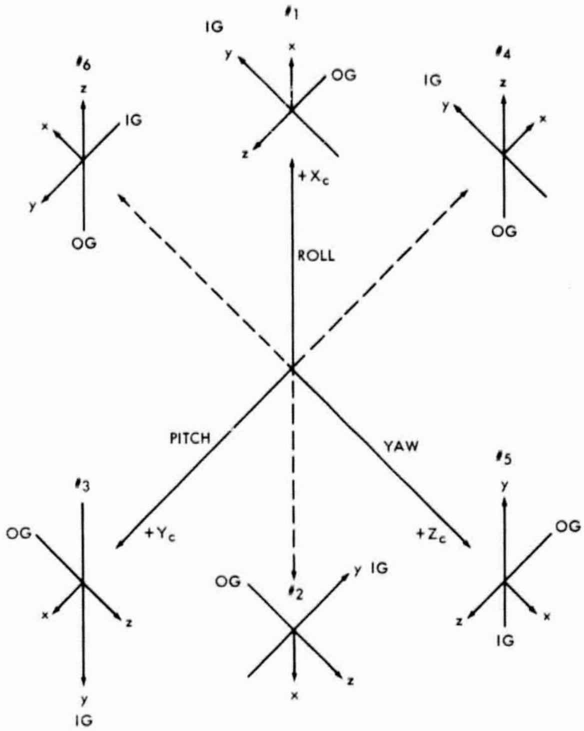


Fig. 1. Control Axis System and Star Tracker Gimbal Locations

are defined such that the zenith position of tracker k lies along the $+x_k$-axis, the inner gimbal axis at zenith[2] coincides with the y_k-axis, and the outer gimbal axis coincides with the z_k-axis. Within the local tracker k coordinate system, the outer and inner gimbal phasing is defined according to the right ascension-declination convention: outer gimbal motion about the z_k-axis is positive counterclockwise, inner gimbal motion about the inner gimbal axis is positive clockwise.[3] These relationships are shown in Fig. 2.

Hence a star in the field of view of tracker k with gimbal angles σ_k, μ_k (outer, inner, resp.) has

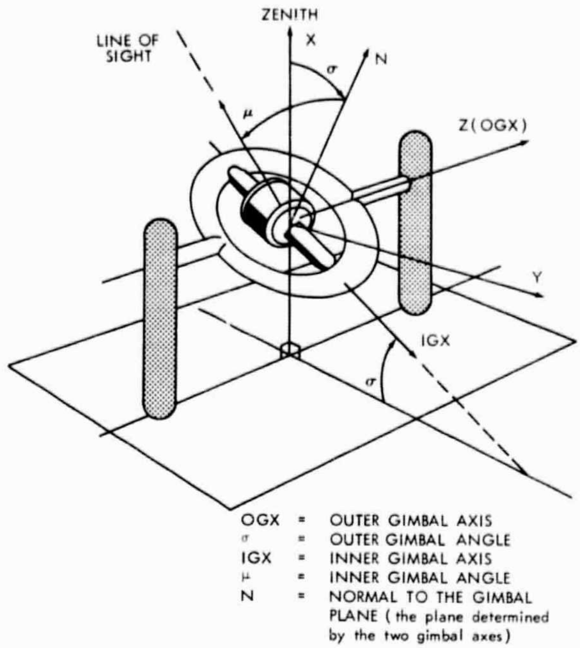


Fig. 2. Gimbal Angles

local coordinates

$$\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix} = \begin{pmatrix} c\sigma_k & s\sigma_k & 0 \\ -s\sigma_k & c\sigma_k & 0 \\ 0 & 0 & 1 \end{pmatrix}^T \begin{pmatrix} c\mu_k & 0 & s\mu_k \\ 0 & 1 & 0 \\ -s\mu_k & 0 & c\mu_k \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} c\sigma_k \, c\mu_k \\ s\sigma_k \, c\mu_k \\ s\mu_k \end{pmatrix} .$$

This transformation is represented by the following coordinate-transformation diagram:

$$\text{(k)} \xrightarrow[z]{\sigma_k} \bigcirc \xrightarrow[-y]{\mu_k} \text{(k*)}$$

As each of the six local coordinate systems is nominally aligned parallel to some control set of axes, the

[2]The inner gimbal rides in the outer gimbal, so that the inner gimbal axis rotates with the outer gimbal. Hence the inner gimbal axis coincides with the y_k-axis only for zero outer gimbal angle.

[3]Phasing is defined in this way for the mathematical model; phasing in the physical spacecraft is slightly different from that defined here.

nominal transformation R_k from the control axis c to the system k has a matrix composed only of 0, ±1:

$$R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \qquad R_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R_4 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad R_6 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Each such transformation is represented by the following diagram:

$$c \xrightarrow{R_k} k$$

C. Mathematical Model of Misalignments

The misalignments modeled were, first, rotational misalignments $d\phi_k$, $d\theta_k$, $d\psi_k$ (taken positive in the conventional right-hand sense) about each of the tracker k coordinate axes x_k, y_k, z_k, resp. These represent an arbitrary misalignment of the tracker k gimbal platform relative to the spacecraft structure as a whole. Making the usual small-angle (first-order) approximations

$$c\alpha \approx 1 , \qquad s\alpha \approx \alpha \left(|\alpha| << 1 \right)$$

the misalignments can be represented by a small-angle rotation matrix

$$I + d\Phi_k = \begin{pmatrix} 1 & d\psi_k & -d\theta_k \\ -d\psi_k & 1 & d\phi_k \\ d\theta_k & -d\phi_k & 1 \end{pmatrix}$$

Second, there were also modeled shifts of the null position in the inner and outer gimbals. A null shift in the outer gimbal cannot be distinguished from a misalignment about the tracker z_k-axis, since the outer gimbal axis is always parallel to the z_k-axis. Hence no separate parameter is necessary to represent this shift.

However, a null shift $d\beta_k$ in the inner gimbal can be separated from a misalignment about the tracker y_k-axis by taking a large outer gimbal angle, since this separates the inner gimbal axis from the y_k-axis. (The inner gimbal axis is parallel to the y_k-axis only when the outer gimbal axis is zero.)

Because $d\beta_k$ masquerades as $d\theta_k$ for zero outer gimbal angle, the sense of $d\beta_k$ has been taken as that of $d\theta_k$, viz., positive in the usual right-hand sense. This is opposite to the sense of the inner gimbal angle itself, which is that of declination (declination is negative in the usual right-hand sense).

There are of course some misalignments present at launch. These misalignments are measured during pre-launch calibration, and are included in the software in the following way. First, the initial rotational misalignments are measured for each tracker and incorporated as an initial constant small-angle rotation matrix $I + d\Omega_k$. Second, the initial inner gimbal null shift is incorporated in such a way that the inner gimbal command issued includes this null shift.

D. Summary

Thus we have defined the following coordinate transformation models for each tracker. In the nominal case, the outer and inner gimbal angles σ_k, μ_k, resp., are computed based on the known attitude of the spacecraft and the known misalignments $d\Omega_k$:

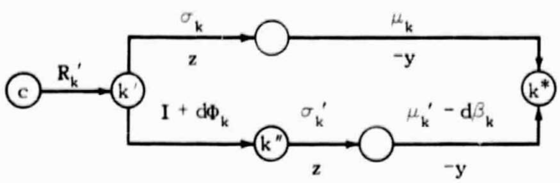
$$c \xrightarrow{R_k} k \xrightarrow{I + d\Omega_k} k' \xrightarrow[z]{\sigma_k} \bigcirc \xrightarrow[-y]{\mu_k} k^*$$

These gimbal angles would nominally point the star-tracker k line-of-sight directly at the target star. Due to unknown misalignments $d\Phi_k$, $d\beta_k$, however, and to the fact that the true spacecraft attitude could be somewhat removed from nominal, the tracker will not in general find the target star at the commanded angles σ_k, μ_k, but rather at measured angles σ_k', μ_k' slightly different from σ_k, μ_k. Hence including misalignments, we have the following situation:

$$c \xrightarrow{R_k} k \xrightarrow{I + d\Omega_k} k' \xrightarrow{I + d\Phi_k} k'' \xrightarrow[z]{\sigma_k'} \bigcirc \xrightarrow[-y]{\mu_k' - d\beta_k} k^*$$

(The negative sign before $d\beta_k$ is due to the fact that its sense is opposite to that of μ_k'.) Thus for a

given star being tracked by a given tracker, we have the following:

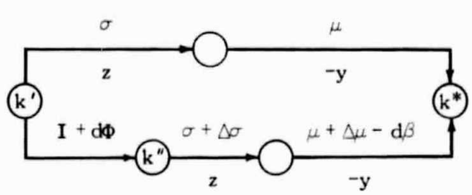


where $R_k' = (I + d\Omega_k) R_k$.

IV. MISALIGNMENT EQUATIONS

A. Known Spacecraft Attitude

First, consider the situation which would prevail if the spacecraft attitude were perfectly known. Suppose that at this attitude a certain startracker k views a fixed star, at nominal gimbal angles σ, μ (outer and inner, resp.). Due to misalignments, however, the star is actually found at measured gimbal angles $\sigma' \equiv \sigma + \Delta\sigma$, $\mu' \equiv \mu + \Delta\mu$:



In coordinate system k^*, the viewed star has coordinates (1,0,0); in coordinate system k', the viewed star has some coordinates $\vec{V}$. Working backwards from system k^* to system k' by either path should lead to the same coordinates $\vec{V}$. Hence we have the equation:

$$\begin{pmatrix} \cos\sigma & -\sin\sigma & 0 \\ \sin\sigma & \cos\sigma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\mu & 0 & -\sin\mu \\ 0 & 1 & 0 \\ \sin\mu & 0 & \cos\mu \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= V = \begin{pmatrix} 1 & -d\psi & d\theta \\ d\psi & 1 & -d\phi \\ -d\theta & d\phi & 1 \end{pmatrix}$$

$$\cdot \begin{pmatrix} \cos(\sigma+\Delta\sigma) & -\sin(\sigma+\Delta\sigma) & 0 \\ \sin(\sigma+\Delta\sigma) & \cos(\sigma+\Delta\sigma) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cdot \begin{pmatrix} \cos(\mu+\Delta\mu-d\beta) & 0 & -\sin(\mu+\Delta\mu-d\beta) \\ 0 & 1 & 0 \\ \sin(\mu+\Delta\mu-d\beta) & 0 & \cos(\mu+\Delta\mu-d\beta) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Now for an arbitrary angle α and a small angle $\Delta\alpha$, we can make the first-order approximations:

$$\cos(\alpha+\Delta\alpha) \approx \cos\alpha - \Delta\alpha\sin\alpha\ ,$$

$$\sin(\alpha+\Delta\alpha) \approx \sin\alpha + \Delta\alpha\cos\alpha$$

The misalignments $d\phi$, $d\theta$, $d\psi$, $d\beta$ are assumed small, and the errors $\Delta\sigma$, $\Delta\mu$ must therefore likewise be small. Hence the right-hand member of the above equation can be put in the form:

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -d\psi & d\theta \\ d\psi & 0 & -d\phi \\ -d\theta & d\phi & 0 \end{pmatrix}\right]$$

$$\cdot \left[\begin{pmatrix} \cos\sigma & -\sin\sigma & 0 \\ \sin\sigma & \cos\sigma & 0 \\ 0 & 0 & 1 \end{pmatrix}\right.$$

$$\left. + \Delta\sigma \begin{pmatrix} -\sin\sigma & -\cos\sigma & 0 \\ \cos\sigma & -\sin\sigma & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

$$\cdot \left[\begin{pmatrix} \cos\mu & 0 & -\sin\mu \\ 0 & 1 & 0 \\ \sin\mu & 0 & \cos\mu \end{pmatrix}\right.$$

$$\left. + (\Delta\mu - d\beta) \begin{pmatrix} -\sin\mu & 0 & -\cos\mu \\ 0 & 0 & 0 \\ \cos\mu & 0 & -\sin\mu \end{pmatrix}\right] \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Retaining only terms to first order in the small angles, we have:

$$\begin{pmatrix} \cos\sigma\cos\mu \\ \sin\sigma\cos\mu \\ \sin\mu \end{pmatrix} \approx \begin{pmatrix} \cos\sigma\cos\mu \\ \sin\sigma\cos\mu \\ \sin\mu \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & -d\psi & d\theta \\ d\psi & 0 & -d\phi \\ -d\theta & d\phi & 0 \end{pmatrix} \begin{pmatrix} \cos\sigma\cos\mu \\ \sin\sigma\cos\mu \\ \sin\mu \end{pmatrix}$$

$$+ \Delta\sigma \begin{pmatrix} -\sin\sigma\cos\mu \\ \cos\sigma\cos\mu \\ 0 \end{pmatrix}$$

$$+ (\Delta\mu - d\beta) \begin{pmatrix} -\cos\sigma\sin\mu \\ -\sin\sigma\sin\mu \\ \cos\mu \end{pmatrix}$$

which leads to

$$\begin{pmatrix} 0 & \sin\mu & -\sin\sigma\cos\mu & \cos\sigma\sin\mu \\ -\sin\mu & 0 & \cos\sigma\cos\mu & \sin\sigma\sin\mu \\ \sin\sigma\cos\mu & -\cos\sigma\cos\mu & 0 & -\cos\mu \end{pmatrix}$$

$$\cdot \begin{pmatrix} d\phi \\ d\theta \\ d\psi \\ d\beta \end{pmatrix} \approx \begin{pmatrix} \sin\sigma\cos\mu & \cos\sigma\sin\mu \\ -\cos\sigma\cos\mu & \sin\sigma\sin\mu \\ 0 & -\cos\mu \end{pmatrix} \begin{pmatrix} \Delta\sigma \\ \Delta\mu \end{pmatrix}$$

Thus if for a particular tracker, one reading of the telemetry σ, μ, $\Delta\sigma$, $\Delta\mu$ is given, one can generate three equations in that tracker's four unknown misalignments $d\phi$, $d\theta$, $d\psi$, $d\beta$. If two readings are given representing different pointings (and in particular, different values of σ), one can generate six equations in the four unknowns, and in general four of these will be independent, so that a unique solution may be obtained. If more than four equations are available, one may obtain a least-squares solution.

B. Operational Considerations

The situation described above prevails only under very artificial circumstances. On OAO-A2 the above procedure was used to obtain initial rough estimates of the post-launch misalignments as follows. There is another startracker, the boresighted startracker, in addition to the gimballed startrackers, mounted on the spacecraft. Its optical axis is aligned nominally along the $+x_c$-axis. This startracker was directed at a suitable star, then commanded to hold the spacecraft in pitch and yaw, while one of the side-looking gimballed startrackers was directed at a suitable star and commanded to hold the spacecraft in roll. The following assumptions are made:

i. The boresighted startracker is aligned to the $+x_c$-axis

ii. The side-looking tracker is aligned in the roll-controlling gimbal

iii. The control system holds the spacecraft fixed

Then one can in theory direct any of the five remaining gimballed startrackers to several stars as outlined in the previous section, and determine the misalignments as described. This procedure was actually used during the early orbital checkout to identify any large misalignments present.

The assumptions made do not hold rigorously, of course, and hence the method has limited precision.

The first assumption, that the boresighted startracker is aligned to the $+x_c$-axis, is not too serious for OAO-A2, for the axis of interest is the WEP optical axis, and the experimenter has the capability of measuring the pitch and yaw deviations of his optics from the boresighted startracker optics. These deviations can then be propagated through all the six gimballed startrackers as artificial misalignments in such a way that the experimenter optics are properly pointed.

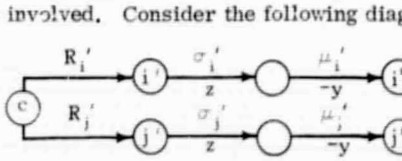
The second assumption, that the particular side-looking tracker chosen accurately controls the roll coordinate, is also not serious for the OAO-A2, for the experiment optics are not roll-sensitive. The roll coordinate is used to maximize solar paddle power output by rolling so as to align the solar paddle as nearly normal to the sunline as possible. But the power output varies like the cosine of the deviation from this optimum roll angle, and hence is not sensitive to first-order variations.

The final assumption mentioned above, that the control system holds the spacecraft fixed, creates a lower bound on the precision obtainable with the method, of about 15 arcseconds. This order of precision has actually been obtained under very tightly constrained operations.

C. Unknown Spacecraft Attitude

The design pointing accuracy of the OAO-A2 under gimballed startracker control was one minute of arc. To achieve this accuracy, gimballed startracker misalignments must be known to at least the same order of magnitude. Hence it is desired to create an alignment estimation technique which does not depend on knowing the spacecraft attitude precisely.

This is accomplished by considering the tracking startrackers by pairs. On the one hand, the angle between two stars individually being tracked by two corresponding startrackers is known precisely from star catalogs. On the other hand, the angle between the tracking startrackers as computed from the gimbal measurements will differ from the true angle, and this discrepancy is assumed to be due to misalignments of the trackers involved. Consider the following diagram:



The computed dot product between the trackers i and j is given by

$$b' = \vec{s}_j{}'^{c} \cdot \vec{s}_i{}'^{c}$$

where $\vec{s}_i{}'^{c}$ are the measured (′) coordinates of star i $(\vec{s}_i)$ in the control coordinate system (^{c}). From the diagram:

$$\vec{s}_i{}'^{c} = R_i{}'^{T} \begin{pmatrix} c\sigma_i' & s\sigma_i' & 0 \\ -s\sigma_i' & c\sigma_i' & 0 \\ 0 & 0 & 1 \end{pmatrix}^{T}$$

$$\cdot \begin{pmatrix} c\mu_i' & 0 & s\mu_i' \\ 0 & 1 & 0 \\ -s\mu_i' & 0 & c\mu_i' \end{pmatrix}^{T} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = R_i{}'^{T} \begin{pmatrix} c\sigma_i' \, c\mu_i' \\ s\sigma_i' \, c\mu_i' \\ s\mu_i' \end{pmatrix}$$

Similarly for tracker j.

On the other hand, the true dot product between the stars is known from the star catalog, and may be expressed as a function of the (unknown) misalignments and the (known) measured angles:

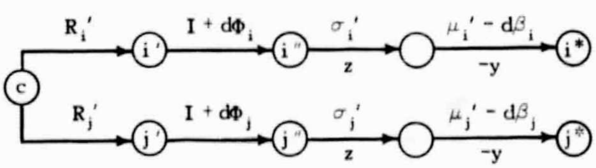


Fig. 3.

Taking the dot product in the i′ coordinate system:

$$b = \vec{s}_j{}^{i'} \cdot \vec{s}_i{}^{i'} .$$

The difference $\Delta b \equiv b - b'$ is a function of the eight unknown misalignments $d\phi_i$, $d\theta_i$, $d\psi_i$, $d\beta_i$, $d\phi_j$, $d\theta_j$, $d\psi_j$, $d\beta_j$, and hence may be expressed to first order in differential form:

$$\Delta b = \left(\frac{\partial \Delta b}{\partial d\phi_i}\right)_0 d\phi_i + \left(\frac{\partial \Delta b}{\partial d\theta_i}\right)_0 d\theta_i + \cdots + \left(\frac{\partial \Delta b}{\partial d\beta_j}\right)_0 d\beta_j$$

$$= \left[\left(\frac{\partial \Delta b}{\partial d\phi_i}\right)_0 \left(\frac{\partial \Delta b}{\partial d\theta_i}\right)_0 \cdots \left(\frac{\partial \Delta b}{\partial d\beta_j}\right)_0\right] \begin{bmatrix} d\phi_i \\ d\theta_i \\ \vdots \\ d\beta_j \end{bmatrix}$$

where the differential coefficients are all evaluated in the nominal state, i.e., assuming zero misalignments. For any given data reading, the discrepancy $\Delta b = b - b'$ is computed from the known coordinates of the stars and the data (the measured angles σ_i', μ_i', σ_j', μ_j'). The differential coefficients are also computed from the data, as follows:

$$\Delta b = b - b' = \vec{s}_j{}^{i'} \cdot \vec{s}_i{}^{i'} - \vec{s}_j{}'^{c} \cdot \vec{s}_i{}'^{c} ,$$

$$\frac{\partial \Delta b}{\partial d\phi_i} = \frac{\partial}{\partial d\phi_i} \left(\vec{s}_j{}^{i'} \cdot \vec{s}_i{}^{i'}\right) = \vec{s}_j{}^{i'} \cdot \frac{\partial \vec{s}_i{}^{i'}}{\partial d\phi_i} .$$

Now

$$\vec{s}_i{}^{i'} = \begin{pmatrix} 1 & d\psi_i & -d\theta_i \\ -d\psi_i & 1 & d\phi_i \\ d\theta_i & -d\phi_i & 1 \end{pmatrix}^{T} \vec{s}_i{}^{i''} .$$

(The coordinate system i″ is that of the misaligned star-tracker i gimbal platform. Cf. Fig. 3.) Hence

$$\frac{\partial \vec{s}_i{}^{i'}}{\partial d\phi_i} = \frac{\partial}{\partial d\phi_i} \begin{pmatrix} 1 & -d\psi_i & d\theta_i \\ d\psi_i & 1 & -d\phi_i \\ -d\theta_i & d\phi_i & 1 \end{pmatrix} \vec{s}_i{}^{i''}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \vec{s}_i{}^{i''} .$$

Evaluating $\vec{s}_i{}^{i''}$ in the nominal state, this becomes

$$\left(\frac{\partial \vec{s}_i{}^{i'}}{\partial d\phi_i}\right)_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \vec{s}_i{}^{i'}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} c\sigma_i' \, c\mu_i' \\ s\sigma_i' \, c\mu_i' \\ s\mu_i' \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -s\mu_i' \\ s\sigma_i' \, c\mu_i' \end{pmatrix} .$$

Hence

$$\left(\frac{\partial \Delta b}{\partial d\phi_i}\right)_0 = \vec{s}_j^{\,i'} \cdot \begin{pmatrix} 0 \\ -s\mu_i' \\ s\sigma_i'\, c\mu_i' \end{pmatrix}$$

$$= \left(R_i'\, R_j'^{\,T}\, \vec{s}_j^{\,j'}\right)^T \begin{pmatrix} 0 \\ -s\mu_i' \\ s\sigma_i'\, c\mu_i' \end{pmatrix}$$

$$= \begin{pmatrix} c\sigma_j'\, c\mu_j' \\ s\sigma_j'\, c\mu_j' \\ s\mu_j' \end{pmatrix}^T R_j'\, R_i'^{\,T} \begin{pmatrix} 0 \\ -s\mu_i' \\ s\sigma_i'\, c\mu_i' \end{pmatrix} .$$

In a similar way,

$$\left(\frac{\partial \Delta b}{\partial d\theta_i}\right)_0 = \vec{s}_j^{\,i'} \cdot \begin{pmatrix} s\mu_i' \\ 0 \\ -c\sigma_i'\, c\mu_i' \end{pmatrix} ,$$

$$\left(\frac{\partial \Delta b}{\partial d\psi_i}\right)_0 = \vec{s}_j^{\,i'} \cdot \begin{pmatrix} -s\sigma_i'\, c\mu_i' \\ c\sigma_i'\, c\mu_i' \\ 0 \end{pmatrix} .$$

Finally,

$$\frac{\partial \vec{s}_i^{\,i'}}{\partial d\beta_i} = \frac{\partial}{\partial d\beta_i} \begin{pmatrix} 1 & d\psi_i & -d\theta_i \\ -d\psi_i & 1 & d\phi_i \\ d\theta_i & -d\phi_i & 1 \end{pmatrix}^T$$

$$\cdot \begin{pmatrix} c\sigma_i' & s\sigma_i' & 0 \\ -s\sigma_i' & c\sigma_i' & 0 \\ 0 & 0 & 1 \end{pmatrix}^T \begin{pmatrix} c\mu_i' & 0 & s\mu_i' \\ 0 & 1 & 0 \\ -s\mu_i' & 0 & c\mu_i' \end{pmatrix}^T$$

$$\cdot \begin{pmatrix} 1 & 0 & -d\beta \\ 0 & 1 & 0 \\ d\beta & 0 & 1 \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -d\psi_i & d\theta_i \\ d\psi_i & 1 & -d\phi_i \\ -d\theta_i & d\phi_i & 1 \end{pmatrix} \begin{pmatrix} c\sigma_i'\, s\mu_i' \\ s\sigma_i'\, s\mu_i' \\ -c\mu_i' \end{pmatrix} .$$

At zero misalignments,

$$\left(\frac{\partial \Delta b}{\partial d\beta_i}\right)_0 = \vec{s}_j^{\,i'} \cdot \begin{pmatrix} c\sigma_i'\, s\mu_i' \\ s\sigma_i'\, s\mu_i' \\ -c\mu_i' \end{pmatrix} .$$

The coefficients

$$\left(\frac{\partial \Delta b}{\partial d\phi_j}\right)_0 , \quad \left(\frac{\partial \Delta b}{\partial d\theta_j}\right)_0 , \quad \left(\frac{\partial \Delta b}{\partial d\psi_j}\right)_0 , \quad \left(\frac{\partial \Delta b}{\partial d\beta_j}\right)_0$$

can be computed using the above formulas, due to the symmetry of i and j, simply by interchanging i and j. Hence the following equation in eight unknowns has been generated:

$$\left[C_{ji} \,\vdots\, C_{ij}\right] \begin{bmatrix} d\phi_i \\ d\theta_i \\ d\psi_i \\ d\beta_i \\ d\phi_j \\ d\theta_j \\ d\psi_j \\ d\beta_j \end{bmatrix} = b - b' \tag{1}$$

where C_{ji}, C_{ij} are 1×4 matrices:

$$C_{k\ell} = \begin{pmatrix} c\sigma_k'\, c\mu_k' \\ s\sigma_k'\, c\mu_k' \\ s\mu_k' \end{pmatrix}^T R_k'\, R_\ell'^{\,T}$$

$$\cdot \begin{pmatrix} 0 & s\mu_\ell' & -s\sigma_\ell'\, c\mu_\ell' & c\sigma_\ell'\, s\mu_\ell' \\ -s\mu_\ell' & 0 & c\sigma_\ell'\, c\mu_\ell' & s\sigma_\ell'\, s\mu_\ell' \\ s\sigma_\ell'\, c\mu_\ell' & -c\sigma_\ell'\, c\mu_\ell' & 0 & -c\mu_\ell' \end{pmatrix}$$

Altogether in the system of the six gimballed startrackers, there are 24 unknowns, and the above Eq. (1) may be regarded as one equation in 24 unknowns. Each pair of tracking startrackers generates one such equation for each data reading. A set of three tracking startrackers taken by pairs generates three such equations, and in general a set of n tracking startrackers generates $\binom{n}{2}$ such equations (some redundant).

Data readings are collected representing many different values of σ, μ for all trackers, and the equations described above are generated. In this way a large

number of equations in the 24 unknown misalignments are generated. This system of equations is then solved in the least-squares sense. The solutions represent the least-squares estimates sought.

The quality of the estimates obtained is controlled by processing several sets of data, so that the consistency of each estimate may be monitored. Several interesting characteristics have appeared as a result of this quality evaluation. Some of these features are discussed in the next section.

V. SPECIAL FEATURES

A. Tracker in or out of Control Loop

There are two tracking modes of startracker operation. In both modes, the startracker is locked onto a star (its guide star), and continuously reads out its error angles "measured-minus-commanded" in both gimbals.

In the usual operational mode, the Auto Track Mode, these error signals are then resolved into components of error about each of the three spacecraft control axes, and averaged with similar components from the other tracking startrackers. The average error signals then control the spinning up or down of inertia wheels to restore the spacecraft to its nominal attitude. It is then clear that the error signals from a tracker operating in the Auto Track Mode by themselves give no information about the misalignments of that tracker relative either to nominal or to any other tracker. The spacecraft takes on attitudes near nominal in response to all the errors seen by all the tracking startrackers in the control loop; these attitudes are away from nominal and hence generate additional errors in all the trackers in the loop, including the perfectly aligned trackers. Hence it is not possible in the Auto Track Mode to separate errors due to misalignment from errors due to other sources simply by observing the errors from one startracker.

Another tracking mode of startracker operation, the Forced Track Mode, is also available. If a startracker is tracking in this mode, its error signals are not mixed into the control loop. (The spacecraft attitude must be controlled by some other means, such as by other gimballed startrackers, by the Rate and Position Sensor (RAPS) accelerometer package, etc.) If attitude reference can be maintained in a sufficiently precise way, then the gimbal error signals read out will actually represent the misalignments of the tracker with respect to the reference coordinates being maintained. If conditions are right, this mode can be used to obtain gross misalignment estimates fairly quickly, as discussed in Section IV.A.

B. Misalignment Reference

The 24 unknown misalignments are not all determined by the estimation procedure. Since only the angles between pairs of startrackers are involved, there is no fixed reference in the body axes to which all the trackers can be aligned. Hence normally startracker #1, the forward-looking tracker, is chosen as a reference and assigned zero rotational misalignments $d\phi_1 = d\theta_1 = d\psi_1 = 0$ (it may still have a nonzero inner gimbal null shift $d\beta_1$). Then the remaining misalignments are estimated by the procedure described. The experimenter has the capability of determining the deviation in pitch and yaw of his optics from the startracker #1 line-of-sight. It has already been noted that the experiment is not roll-sensitive, so from that point of view it is permissible to leave the roll reference arbitrary.

C. Separation of $d\beta$ from $d\theta$

It has turned out to be more difficult than anticipated to separate $d\beta$ from $d\theta$, that is, to decide what portion of the error appearing about the inner gimbal axis is due to null shift $d\beta$ in the inner gimbal and what portion is due to rotational misalignment $d\theta$ of the gimbal platform about the tracker y-axis.

From Eq. (1), assuming that the other misalignments have been determined, the pertinent equation for some tracker i has the form

$$\vec{s}_j^{\,i'} \cdot \begin{pmatrix} s\mu_i' \\ 0 \\ -c\sigma_i' \, c\mu_i' \end{pmatrix} d\theta_i + \vec{s}_j^{\,i'} \cdot \begin{pmatrix} c\sigma_i' \, s\mu_i' \\ s\sigma_i' \, s\mu_i' \\ -c\mu_i' \end{pmatrix} d\beta_i = \text{constant} .$$

When $\sigma_i' = 0$, the inner gimbal axis and the y_i-axis coincide:

$$c\sigma_i' = 1 , \qquad s\sigma_i' = 0 ,$$

$$\vec{s}_j^{\,i'} \cdot \begin{pmatrix} s\mu_i' \\ 0 \\ -c\mu_i' \end{pmatrix} \left(d\theta_i + d\beta_i \right) = \text{constant}$$

Thus only the sum $d\theta_i + d\beta_i$ can be determined. To separate $d\beta_i$ from $d\theta_i$, it is necessary to take large values of σ_i', so that the coefficients

$$\vec{s}_j^{\,i'} \cdot \begin{pmatrix} s\mu_i' \\ 0 \\ -c\sigma_i' \, c\mu_i' \end{pmatrix}$$

and

$$\vec{s}_j^{\,i'} \cdot \begin{pmatrix} c\sigma_i' \, s\mu_i' \\ s\sigma_i' \, s\mu_i' \\ -c\mu_i' \end{pmatrix}$$

are sufficiently distinct.

If σ_i' is small, then the difference between these coefficients can be approximated to first-order:

$$\vec{s}_j^{\,i'} \cdot \begin{pmatrix} s\mu_i' \\ 0 \\ -c\sigma_i' \, c\mu_i' \end{pmatrix} - \vec{s}_j^{\,i'} \cdot \begin{pmatrix} c\sigma_i' \, s\mu_i' \\ s\sigma_i' \, s\mu_i' \\ -c\mu_i' \end{pmatrix}$$

$$= \vec{s}_j^{\,i'} \cdot \begin{pmatrix} s\mu_i' - c\sigma_i' \, s\mu_i' \\ -s\sigma_i' \, s\mu_i' \\ -c\sigma_i' \, c\mu_i' + c\mu_i' \end{pmatrix}$$

$$\approx \vec{s}_j^{\,i'} \cdot \begin{bmatrix} s\mu_i' \, (\sigma_i')^2 \\ -s\mu_i' \, (\sigma_i') \\ c\mu_i' \, (\sigma_i')^2 \end{bmatrix} = \sigma_i' \, \vec{s}_j^{\,i'} \cdot \begin{pmatrix} \sigma_i' \, s\mu_i' \\ -s\mu_i' \\ \sigma_i' \, c\mu_i' \end{pmatrix}$$

which is first order in the small angle. If in addition, μ_i' is small, this becomes approximately

$$\sigma_i' \, \vec{s}_j^{\,i'} \cdot \begin{pmatrix} \sigma_i' \, \mu_i' \\ -\mu_i' \\ \sigma_i' \end{pmatrix}$$

which is second order in small angles. Now in the course of collecting data, a considerable portion of the data will have both gimbal angles small enough to cause difficulty in separating these parameters, since the procedure followed by the computer in selecting guide star patterns favors small gimbal angles.

D. Optimal Estimation

The above feature points out one of the limitations of least-squares estimation in this application. Functional dependence between parameters is not taken into account. Furthermore the data are assumed to be representative of all the parameters equally. Due to the manner in which the data are collected, this requirement cannot be assured. For these reasons, it may be necessary in the continuing development of this estimation system to perform minimal variance estimation on the data rather than simple least squares. Most of the analysis for such a procedure can be found in Ref. 1.

VI. RESULTS

A. Launch-induced Misalignments

Table 1 lists the misalignments estimated to have been caused by launch stresses. The figures given are the differences between the prelaunch calibration measurements and the early in-orbit estimates. These values were determined by a variety of techniques and data fits, and are given relative to startracker #1.[4] The entries in Table 1 are given to the nearest 0.2 arcminute although their accuracy is probably not better than 0.4 arcminute.

Table 1. Launch-induced Misalignments (arcmin)

Tracker #	Misalignment Parameter			
	$d\phi$	$d\theta$	$d\psi$	$d\beta$
1	0	0	0	0
2	0.4	0.4	-1.0	1.4
3	-5.6	3.0	-1.0	0
4	-1.6	-3.0	-2.8	0
5	-0.6	-0.2	1.0	0.8
6	-3.0	-0.8	-5.6	0

These are significant misalignments. Allowing for 0.4 arcminute error in the entries in Table 1, and even assuming a complete inability to separate $d\beta$ from $d\theta$, the rotational misalignments exceed 2.2 arcminutes rms and 5 arcminutes maximum.

B. Thermal-induced Misalignments

The misalignments given in Table 1 produced exceptionally good pointings in the early orbits, with gimbal errors consistently down in the 10-20 arcsecond range. But when operations with the alternate experimenter (SAO) commenced, the pointings degraded markedly. The degradation is due in some part to

[4] A shift in the pointing of tracker #1 relative to the experiment optics has been removed from the misalignments reported in Table 1, although it is included in the misalignment table for spacecraft operations.

thermal fluctuation. In the early orbits, the operations staff was gingerly in their movements of the spacecraft and tended to operate in a restricted region of the sky. Over a period of several orbits, the misalignments stabilized at or near the values given in Table 1 above.[5]

The SAO experimenter operates out of the opposite end of the spacecraft from WEP, however. Since both stay well away from the sun, this tends to expose the opposite extremities of the spacecraft to the sun's rays. The thermal bending which then takes place is felt to contribute substantial additional misalignment to that given in Table 1. Hence it was deemed advisable to develop an additional (B) set of misalignments for SAO operations. The differences B-A between this modified set and the original (A) set are given in Table 2. These are considered to be in some part thermal-induced misalignments.

Table 2. Misalignment Differences B-A (arcmin)

Tracker #	Misalignment Parameter			
	$d\phi$	$d\theta$	$d\psi$	$d\beta$
1	0.4	1.0	0	0.4
2	-1.8	-0.2	-1.6	1.4
3	0.8	-1.2	-1.8	0.4
4	0.2	3.0	0.6	-1.6
5	-0.8	-0.8	0.2	-1.8
6	0	0.8	0.8	1.2

It should be noted that these B misalignments have not succeeded in reducing the gimbal errors to those experienced in the early orbits. The errors currently run in the 30-50 arcsecond range. One explanation for this may be that the experimenter is currently ranging over a much larger region of the celestial sphere due to a spacecraft problem which is not of concern here. It may be that the thermal fluctuations due to this activity are contributing substantial variation in the misalignments.

The misalignment differences given in Table 2 may be conservatively characterized as were those in Table 1. Allowing for an error of 0.8 arcminute in these entries, and not separating $d\beta$ from $d\theta$, the figures still represent rotational misalignments in excess of 0.6 arcminutes rms and 1.6 arcminutes maximum.

C. Conclusions

Significant gimballed startracker misalignments on the order of 2 to 5 arcminutes due to launch stresses and 0.5 to 1.5 arcminutes due in part to thermal stresses have been estimated on the OAO-A2 using techniques described in this report. This magnitude of misalignments, if uncorrected, would have represented a substantial limitation to high-precision pointing under gimballed startracker control. Using rather crude data collection and estimation techniques, however, misalignment parameters have been obtained of sufficient precision to maintain (and in some constrained operations, to improve on) the OAO-A2 design pointing accuracy of one arcminute. Using improved data collection techniques and optimal misalignment estimation, a theoretical pointing capability under gimballed startracker control of close to one-half arcminute in constrained operations is indicated.

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3. Mathematical Analysis of a Proposed Technique for Calibrating Gimballed Startracker Misalignments, Goddard Space Flight Center OAO-SCPS Technical Memorandum T-68-28, R. desJardins, October 28, 1968.

[5]The thermal inertia of the spacecraft is such that light-dark variation of misalignments during one revolution is negligible.